Solution to
2021~2022 International Mathematics Assessment for Schools
Round 1 of Upper Primary Division

1. Twelve players compose a basketball team. Each player must select a jersey number from 4, 5, 6, ⋯, 14 and 15, where two players cannot choose the same number. If Alvin is a member of the basketball team and it is known that the sum of the jersey numbers of all his other teammates is 106, what is his jersey number?

(A) 6 (B) 8 (C) 10 (D) 12 (E) 14

【Suggested Solution】

Since the players select from numbers 4 to 15, its sum is $4+5+6+...+15=114$.
The sum of the remaining numbers is 106, then Alvin's number is $114-106=8$.

Answer：(B)

2. Let ★ be a positive integer such that $(★-2)\times(★+2)=2021$, what is the value of ★?

(A) 42 (B) 43 (C) 44 (D) 45 (E) 47

【Suggested Solution 1】

Since we know that $2021=43\times 47$, then it easy to see that $★=45$.

【Suggested Solution 2】

Observe that the equation is in the form $(★-2)\times(★+2)=★\times★-2\times 2=2021$, simplifying we get $★\times★=2021+4$, which yields $★\times★=2025$. Since ★ is positive integer, we have $★=45$.

Answer：(D)

3. Teacher Josie brought a box of cookies for her Kindergarten students. If she allocates 6 cookies to each student, there would be 5 cookies left. But when she gives each student 7 cookies, it will be 8 cookies short. How many students are there in her class?

(A) 11 (B) 13 (C) 15 (D) 17 (E) 19

【Suggested Solution】

From the given information, the total cookies she gave each student 7 cookies difference with the total cookies she gave each student 6 cookies is $5+8=13$ cookies. Hence, there are $13\div(7-6)=13$ students in teacher Josie's class.

Answer：(B)

4. Among the choices, which is **NOT** a prime number?

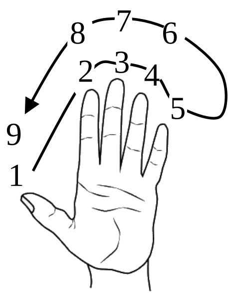
(A) 2 (B) 7 (C) 37 (D) 337 (E) 3337

【Suggested Solution】

It is obvious that 2, 7, 37, 337 are all prime numbers, but $3337=47\times 71$ is not.

Answer：(E)

5. Let's play a game where we start counting from the thumb with the number 1, the index finger is 2, the middle finger is 3, the ring finger is 4 and the little finger is 5, then after which we count in backward order, where the ring finger is 6, the middle finger is 7, the index finger is 8, and the thumb is 9; then we count in forward order again, and we keep on counting in this manner, as shown on the diagram. Which finger will it land into when I count to 2021?



（A）thumb （B）index （C）middle （D）ring （E）little

【Suggested Solution】

Obviously, solving this problem is not relying on counting from finger to finger, but finding out the pattern. From the given diagram, we know that 8 numbers form a cycle, so no matter how big the number is, we only need to divide by 8 first, and after knowing the remainder and starting counting the remainder, then we will know the number is located in which finger.

Divide 2021 by 8, and the final remainder is 5. Just simply count the final count to the little finger.

Answer：(E)

6. In the diagram below, the four unit squares with letters "I", "M", "A" and "S" are rearranged according to the following conditions:

I	**M**
A	**S**

a. The square containing "S" is now on the top row.
b. The square containing "S" remains farther right than "I" in the diagram.
c. The square containing "S" will shares a horizontal side with the square containing "A"
d. Only one of the squares is not moved.

Which of the following diagrams is the new diagram?

(A)

I	**S**
A	**M**

(B)

S	**I**
A	**M**

(C)

S	**M**
A	**I**

(D)

I	**S**
M	**A**

(E)

M	**S**
I	**A**

【Suggested Solution】

From the condition **a**, we have two cases:

S	

or

	S

Then by the condition **b**, we know the former case is impossible, hence we have:

I	**S**

or

	S
I	

Then by the condition **c**, we have:

I	**S**
	A

or

	S
I	**A**

TThen by the condition **d**, the new diagram is

I	**S**
M	**A**

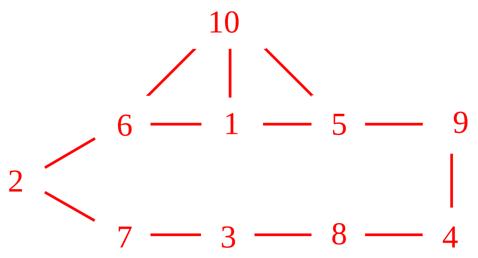
Hence the answer is D.

Answer：(D)

7. At most how many numbers can you choose from the set {1, 2, …, 10} such that the positive difference of any two chosen numbers is not 4, 5 or 9?
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

【Suggested Solution】

Write down the ten numbers and connect any two numbers with differences in {4, 5, 9}, one can get the graph on the right.



If 10 is selected, then 1, 5, 6 are not. Since adjacent numbers can not be both selected, at most three of 2, 7, 3, 8, 4, 9 are selected, giving 4 selected numbers.
If 10 is not selected, then at most 4 among the remaining 9 numbers are chosen.
In summary, there are at most 4 numbers chosen, for example, {1, 2, 3, 4}.

Answer：(A)

8. Eric rolls two standard six-sided dice. How many ways are there for him to get a sum of 6 or 7 or 8?
(A) 7 (B) 9 (C) 14 (D) 15 (E) 16

【Suggested Solution】

We use our notation as (x,y) – where x is the value of the first dice and y is the value of the second dice.
To have a sum of 6, we have the following cases: (1, 5), (5, 1), (2, 4), (4, 2) and (3, 3) – 5 ways.
To have a sum of 7, we have the following cases: (1, 6), (6, 1), (2, 5), (5, 2), (3, 4) and (4, 3) – 6 ways.
To have a sum of 8, we have the following cases: (2, 6), (6, 2), (3, 5), (5, 3) and (4, 4) – 5 ways.
In total, there are $5+6+5=16$ ways.

Answer：(E)

9. Let $\overline{ab}$ be a two-digit number. Now, swap its digits to form another two-digit number, multiply it by 5 and take the remainder of the result when divided by 9. If the resulting number is 5, then how many different $\overline{ab}$ exist?
(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

【Suggested Solution】

The reminder of $5\times\overline{ba}$ when divided by 9 is 5. Therefore the reminder of $a+b$ when divided by 9 is 1. So $a+b=1$ or 10.

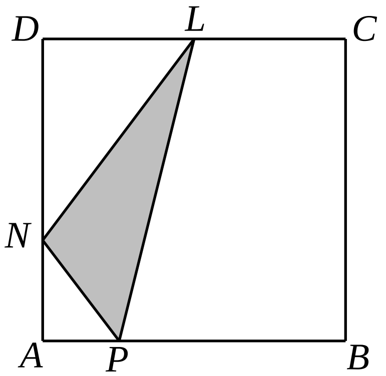
If $a=1$, then $b=0$ or 9. The former case is impossible as $\overline{ba}$ is also a 2-digit number. So $b=9$.

If $a=2$, then $b=8$. If $a=3$, then $b=7$. If $a=4$, then $b=6$. If $a=5$, then $b=5$. If $a=6$, then $b=4$. If $a=7$, then $b=3$. If $a=8$, then $b=2$. If $a=9$, then $b=1$.

Altogether 9 possibilities.

Answer：(C)

10. In the diagram below, *ABCD* is a square with side length 12 cm. Point *P* is on side *AB* such that *AP*:*PB* = 1:3, point *L* is on side *CD* such that $CL=DL$ and point *N* is on side *DA* such that *DN*:*NA* = 2:1. What is the area, in cm^2, of the triangle *PNL*?



(A) 24 (B) 30 (C) 48

(D) 90 (E) 144

【Suggested Solution】

Since *AP*:*PB* = 1:3, $AP=12\times\frac{1}{4}=3$ cm and $BP=12\times\frac{3}{4}=9$ cm.

Since $CL=DL$, $CL=DL=12\times\frac{1}{2}=6$ cm.

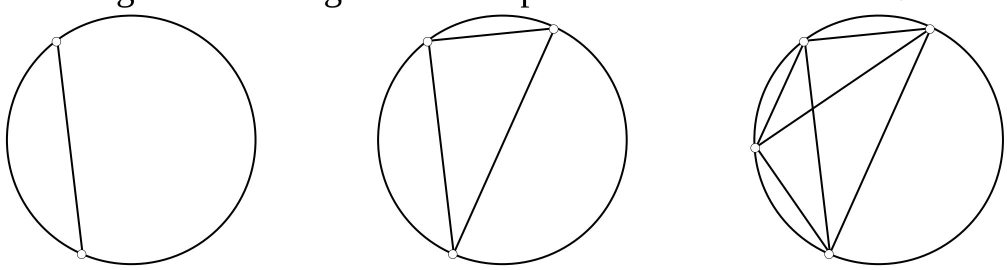
Since *DN*:*NA* = 2:1, $AN=12\times\frac{1}{3}=4$ cm and $DN=12\times\frac{2}{3}=8$ cm.

Then the area of right triangle *APN*, *DNL* and trapezoid *APLD* is $\frac{4\times3}{2}=6$ cm^2, $\frac{6\times8}{2}=24$ cm^2 and $\frac{(3+6)\times12}{2}=54$ cm^2, respectively.

Hence, the area of triangle *PNL* is $54-6-24=24$ cm^2.

Answer：(A)

11. The three circles below have some number of points on their circumference. Connect all the points on the same circle using straight lines and count the number of regions these segments have partitioned the circle into.



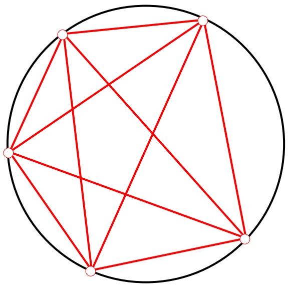
2 points, 2 regions. 3 points, 4 regions. 4 points, 8 regions.

If there are 5 points on the circumference of a circle, how many regions have been partitioned at the most?

(A) 10 (B) 12 (C) 15 (D) 16 (E) 20

【Suggested Solution】

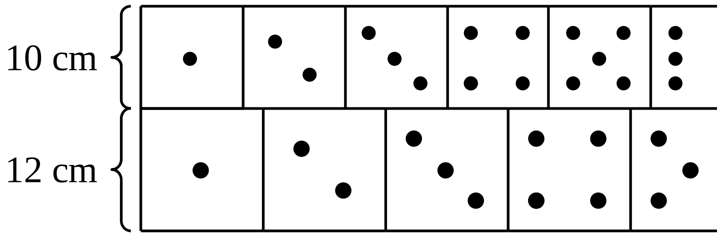
Let any three segments not meet at a same point then we can get at most 16 regions as the diagram shown.

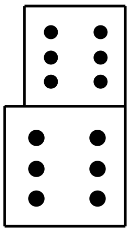


5 points,
16 regions.

Answer：(D)

12. The length of the edge of the small and big dice are 10 cm and 12 cm, respectively. All the big dice are arranged side by side at the lower level, while all the small dice are stacked above the big dice and are also placed side by side at the upper level as shown in the diagram below on the left. The front faces of each row of dice show the same sequence of points: 1, 2, …, 6, 1, 2, …, 6, 1, 2, … When both the small and big dice show the same "6-point" face and their right edge is aligned for the first time (as shown on the diagram below on the right). In what position, when counting starting from the far left, is the big dice?

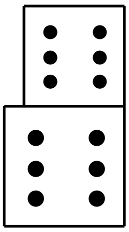




(A) 10 (B) 20 (C) 30 (D) 60 (E) 72

【Suggested Solution】

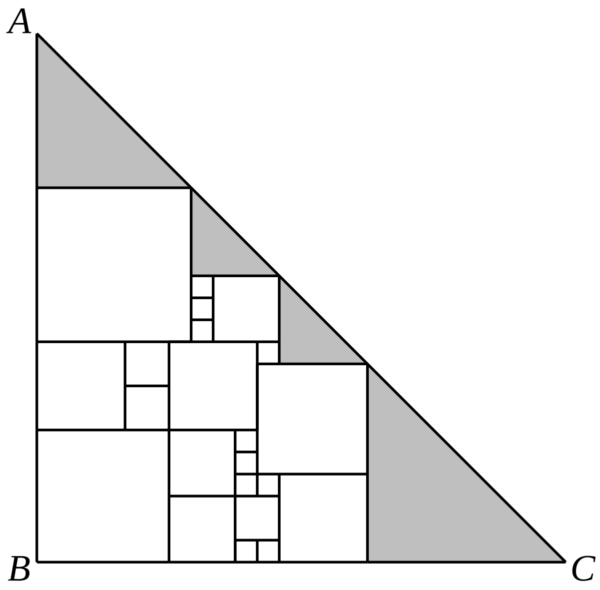
The right sides of the "6-point" front faces of the two dice are aligned must be arranged as shown below.



We know the total length of the first six dice whose side length of 10 cm is 60 cm, and the total length of the first six dice whose side length of 12 cm is 72 cm. Since the LCM of 60 and 72 is 360, and since $360 \div 12 = 30$, indicate the right sides of the two dice of different side length will align on the first time when the big dice is on the 30th dice.

Answer：(C)

13. In the diagram below, ABC is an isosceles right-angled triangle, where $AB=BC=8$ cm. There are various squares of different sizes inside it, while the other triangular parts are shaded. What is the total sum of the areas, in cm^2, of the shaded region?



（A）8　　（B）9　　（C）16　　（D）32　　（E）81

【Suggested Solution】

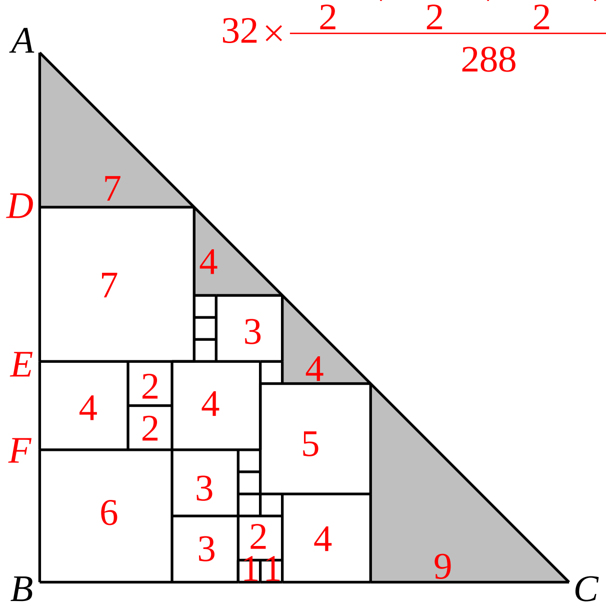
By carefully observing the known diagrams, the length of each part can be determined in their respective order. In this way, the side length of all squares inside triangle ABC can be calculated**.** Suppose the smallest square below has a side length of 1 unit, then the side length of each square are indicate on the diagram below.

Hence we have $AB=AD+DE+EF+FB=7+7+4+6=24$ units, and the leg length of four shaded isosceles right-angled triangles are 7 units, 4 units, 4 units and 9 units, respectively.

We have the area of $\triangle ABC$ is $24\times 24\div 2=288$ square units or $8\times 8\div 2=32$ cm^2.

Therefore, the sum of all the shaded areas in the given diagram is

$$32\times\frac{\frac{7\times 7}{2}+\frac{4\times 4}{2}+\frac{4\times 4}{2}+\frac{9\times 9}{2}}{288}=\frac{32\times 81}{288}=9\,cm^2.$$



Answer：（B）

14. In the given grid, an ant begins at the "Start" cell and in each move, it can go either one step right or down to any adjacent cell with common side until it reaches the "End" cell. If in each move, it gets the number in the square, then what is the smallest sum the ant can get?

(A) 91 (B) 94 (C) 95
(D) 103 (E) 109

Start 	18	15	13	14
20	21	12	26	21
17	7	11	19	10
6	8	17	24	11
10	20	4	19	**End**

【Suggested Solution】

We filled the smallest sum that can reach the square on the grid, as shown in the grid below left.

Start	18	33	46	60
20	39	45	71	81
37	44	55	74	84
43	51	68	92	95
53	71	72	91	End

Start	18	15	13	14
20	21	12	26	21
17	7	11	19	10
6	8	17	24	11
10	20	4	19	End

Then we find the smallest sum to reach the finish is 91, the path as grid shown above right.

Answer：(A)

15. How many ways can we arrange two 1's, two 2's and two 3's in a row such that there is at least one "2" between two 1's, at least one "3" between two 2's and at least one "1" between two 3's?

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

【Suggested Solution 】

First forget about the 3's, there are several cases about relative position of two 1's and two 2's.

(1) 1221. Then one "3" should be between two 2's and the other "3" should be at the front or the last, giving two solutions 312321 or 123213.

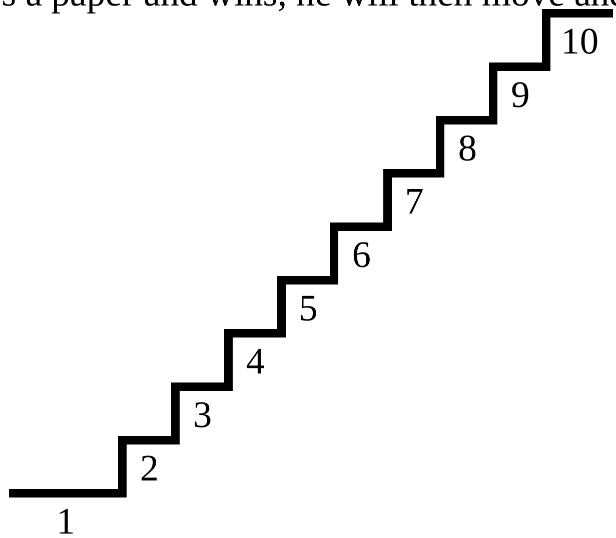
(2) 1212. Look at the two places of x in 12x1x2. At least one of the places with x has a "3", but no place of x should have two 3's. We have three subcases: 12312, 12132 or 123132. Continue talking about the next "3" for the first two subcases. Totally we have 5 solutions: 312312, 123123, 312132, 132132, 123132.

(3) 2121. These will give solutions in one to one correspondence with case (2) by reversing the order, so totally 5 cases.

The answer is $2+5+5=12$.

Answer：(C)

16. Alan and Benjie use the staircase shown in the diagram below to play a game where the goal is to reach a certain level on the staircase first. At the beginning, both of them will start in step 1, and in each turn, they will be playing the “Rock, Paper and Scissors” game to move around the staircase. The winner in each game gets to move 4 consecutive steps upward (or downward or a combination of both) for showing a rock; 5 consecutive steps for showing scissors and 6 consecutive steps for showing a paper. So, for example, when someone reaches step 10, he must go down to the step 9 and so on, finally returning to the step 1 and then goes back up doing the same procedure again until somebody wins. To illustrate further, when a person shows a rock on the first game and wins and then in the second game shows a paper and wins, he will then move and land on step 9.



Since Alan is currently on step 1 of the stairs, what is the least number of times he has to win in order to reach step 2? Note: If a game is a tie (where both players show the same hand, i.e. paper and paper), then they don’t move.

（A）2　　（B）3　　（C）4　　（D）5　　（E）8

【Suggested Solution】

Let us find the possible combination of the game, in order to answer the problem correctly.

Since Alan stopped on the level 2. If the answer just depends on one game, then it is impossible. Therefore, it must be from the level 10 moving down to the level 2. There are 9 move-up steps and 8 move-down steps, with a total of 17 steps. Now, let us use the sum of 4 steps, 5 steps and 6 steps to express 17. So, we have $4+4+4+5=17$ or $6+6+5=17$, it is either one of the two, consider Alan win at least 3 times, which will be Alan won 1 time with scissors and 2 times with paper.

Answer：（B）

17. The following table shows the results of the math test of eight students *A*, *B*, *C*, *D*, *E*, *F*, *G* and *H*. The perfect score is 100 marks and their average score is 64 marks. It is known that student *F* is the top scorer of the math test and his score is twice that of another student, what is the score of student *C* ?

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
74	48		90	33		60	78

（A）33　　（B）37　　（C）39　　（D）45　　（E）96

【Suggested Solution】

Since the average score is 64 points, so their total scores are $64\times 8=512$ points, but the total given scores of the six students is $74+48+90+33+60+78=383$ points. Then, the sum of the scores of student *C* and student *F* is $512-383=129$ points. From the given chart, we know the scores of students *A*, *D*, *G* and *H* are more than 50 points, it follows the score of student *F* may be twice that of one of the students: *B*, *C* and *E*. We also know that twice the score of student *E* is $33\times 2=66$ points, and 66 is not the highest score. the score of student *F* is twice the score of student *C*, then the score of student *F* is $129\div 3\times 2=86$, is not the highest score. It follows, the score of student *F* is twice the score of student *B*, that is $48\times 2=96$ points, it follows the score of student *C* is $129-96=33$ points. From this, it can be seen that the scores of 8 students are as shown in the following table.

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
74	48	33	90	33	96	60	78

Answer：(A)

18. There are five two-digit integers *A*, *B*, *C*, *D* and *E* where $A>B>C>D>E$, such that the average of the five numbers is 33, the average of the four largest numbers is 34 and the average of the four smallest numbers is 31. If it is known that *C* is an even number, what is the value of integer *B*?

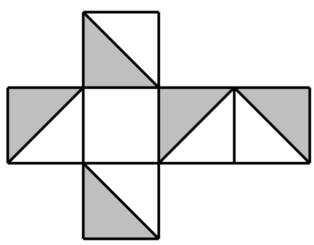
(A) 29　(B) 30　(C) 31　(D) 32　(E) 33

【Suggested Solution】

From the given information, we know the average of the five integers is 33, so their sum is $33\times 5=165$. The average of the four large numbers is 34, then the sum of those four numbers is $34\times 4=136$. It follows the smallest number *E* is $165-136=29$.
Similarly, the sum of the smallest 4 numbers is $31\times 4=124$, this implies the largest number *A* is $165-124=41$. Then the sum of the three numbers locate in the middle is $165-41-29=95$. Furthermore, $3B>B+C+D=95$, then $B\geq 32$. We also have $95=B+C+D>3D$, then $31\geq D$.
When $D=31$, then $B+C=95-31=64>2C$, then $32>C$, since *C* is an even number, we must have $30\geq C$, this is a contradiction.
When $D=30$, then $B+C=95-30=65>2C$, then $32\geq C$, since *C* is an even number, we must have $C=32$ and $B=65-32=33$.

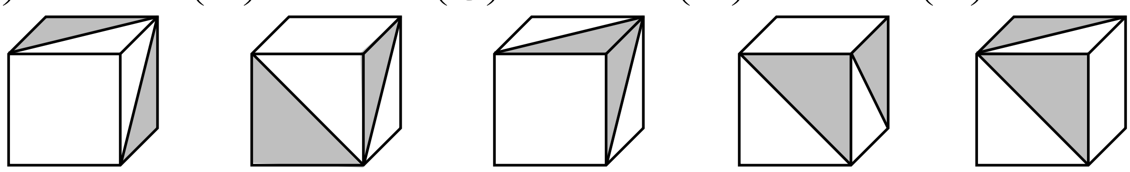
Answer：(E)

19. The diagram on the right shows the net of a cube. Among the following options, which produces the same net?



Net of the cube

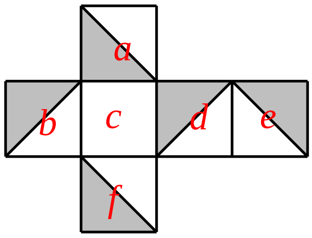
(A)　(B)　(C)　(D)　(E)



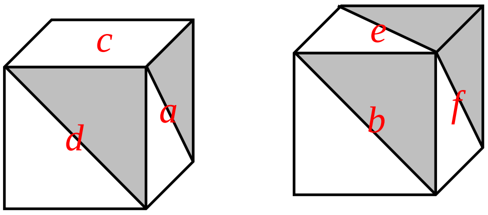
【Suggested Solution】

The answer is D. Focus on the vertex that three faces of the cube intersected. Comparing with the net of cube.

At each vertices of the white square must met at least one shaded triangle, so not (A). The shaded triangles have at least one common side with top white square if met other shaded triangle then they must met at right-angle vertex, so not (B) and (E) . Both shaded triangles have common sides with white square should not have common sides, so not (C).

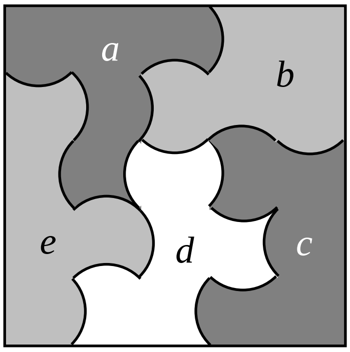


Net of the cube



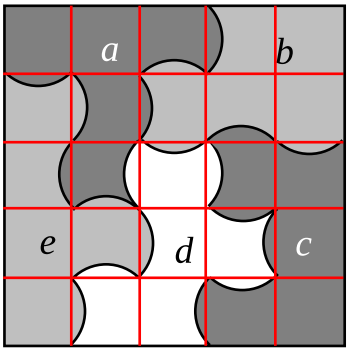
Answer：(D)

20. The diagram below is a square that is divided into five parts. Which two of them have the same perimeter?



(A) *a* and *c* (B) *a* and *d* (C) *c* and *e* (D) *c* and *b* (E) *c* and *d*

【Suggested Solution】



Add the grid as the diagram shown. Then
the perimeter of part *a* is 4 unit plus 8 quarter unit circle,
the perimeter of part *b* is 3 unit plus 6 quarter unit circle,
the perimeter of part *c* is 5 unit plus 7 quarter unit circle,
the perimeter of part *d* is 2 unit plus 10 quarter unit circle,
the perimeter of part *e* is 5 unit plus 7 quarter unit circle.
Hence, part *c* and *e* have the same perimeter.

Answer：(C)

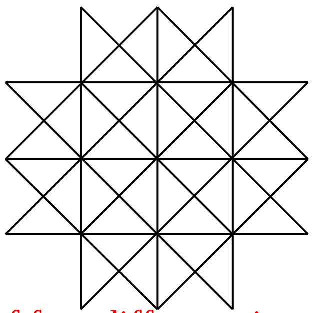
21. Vendor Peter sells red and green apples, where both quantities are the same. He initially plans to sell the apples in the following manner: three red apples for 1 dollar and two green apples for 1 dollar. But to make things easier, he decided to sell them altogether and sells any five apples for 2 dollars. After selling out all his apples, he realizes that the strategy to sell the apples together earns 7 dollars less than his original plan. How many apples of each colour does Peter have at the beginning?

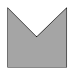
【Suggested Solution】

According to the selling prices, the price of a red apple is $\frac{1}{3}$ dollars and the price of a green apple is $\frac{1}{2}$ dollars, and after putting them together, the price of an apple is $\frac{2}{5}$ dollars. Put an red apple and a green apple as one pair, then selling each pair of apple he will earns $\frac{1}{3}+\frac{1}{2}-\frac{2}{5}\times 2=\frac{10}{30}+\frac{15}{30}-\frac{24}{30}=\frac{1}{30}$ dollars less. Now, he earns 7 dollars less, hence, each colour of apple has $7\div\frac{1}{30}=210$.

Answer：210

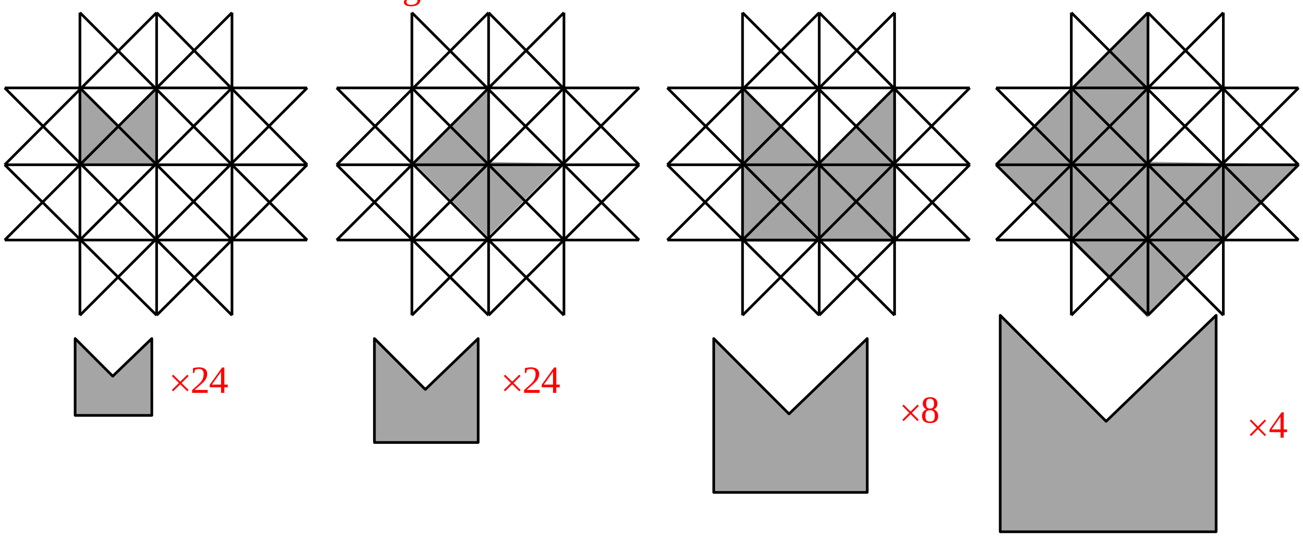
22. How many Mitra-like shapes (M-shapes) of all possible sizes and orientations can be found within the grid below? All M-shapes must be exactly similar to that shown in the small illustration just next to the grid. (Note: an M-shape is a square with a triangular quarter removed.)





【Suggested Solution】

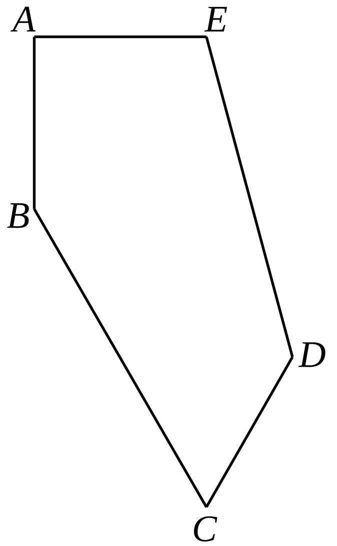
In the grid you can find $24+24+8+4=60$ M- shapes of four different sizes; they are shown within the four grids below.

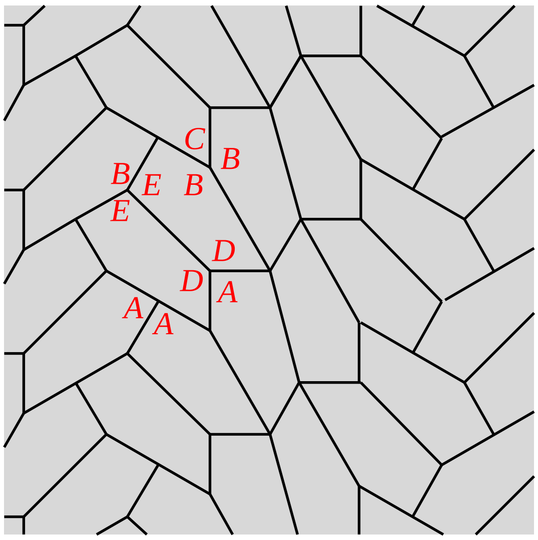


Note that the shapes have different orientation in the grid. The above numbers show total numbers of copies of each particular M-shape hidden in the grid.

Answer：060

23. As show in the diagram below, we use some number of pentagons, that is identical to *ABCDE* to fill the plane below. What is the angle measure, in degrees, of $\angle ABC$?



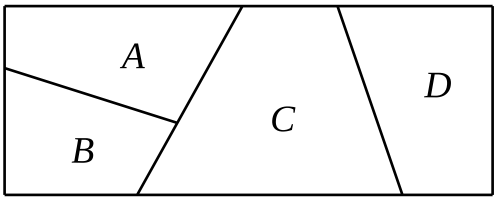


【Suggested Solution】

We notice that twice of $\angle A$ is $180°$, so $\angle A = 90°$. Twice of $\angle D$ and $\angle A$ is a whole $360°$, so $\angle D = \dfrac{360° - 90°}{2} = 135°$. We also find that $\angle C$ and twice of $\angle B$ is a whole $360°$, that is, $\angle C + 2\angle B = 360°$. Similarly, $\angle B + 2\angle E = 360°$. We also have the sum of all angles is $540°$, so $\angle B + \angle C + \angle E = 315°$. Solve these equations, we get $\angle B = 150°$, $\angle C = 60°$ and $\angle E = 105°$.

Answer：150

24. In the figure below, we use four colours, namely red, white, yellow or blue, to paint regions *A*, *B*, *C* and *D* such that any two regions with a common boundary must have different colours. How many colouring methods are there?



【Suggested Solution 1】

First, let us paint red the area *A*. As long as we understand the colouring method, even if the area *A* is painted with other colours, the same situation occurs. When it is quadrupled, it will be the entire painting method. So, the area of *D* can also be painted red, but area *B* and area *C* must be painted in other colours. Possible scenarios for the investigation are as follows.

B(white) → *C* (yellow), *C* (blue)

B(yellow) → *C* (white), *C* (blue)

B(blue) → *C* (white), *C* (yellow)

There are 6 colouring methods.

When the area *D* is painted with a colour other than red, then area *D* and area *B* can be painted in the same or different colours. If it is the same colour, then area *B* and area *C* are

B(white) → *C* (yellow); *C* (blue)

B(yellow) → *C* (white); *C* (blue)

B(blue) → *C* (white); *C* (yellow)

There are also 6 colouring methods. And when painting the area *D* and area *B* with different colours

B(white) → {*C* (yellow), *D* (blue)}; {*C* (blue), *D* (yellow)}

B(yellow) → {*C* (white), *D* (blue)}; {*C* (blue), *D* (white)}

B(blue) → {*C* (white), *D* (yellow)}; {*C* (yellow), *D* (white)}

There are also 6 colouring methods.
Therefore, when area *A* is painted red, there are 18 colouring methods, and when area *A* is painted white, yellow or blue, there will be the same colouring methods.
Therefore, there are a total of $18\times4=72$ colouring methods.

【Suggested Solution 2】

Since any two of areas of *A*, *B* and *C* have common boundary, hence the must colouring 3 different colours. Area *D* have no common boundary with areas of *A* and *B*, hence it can painting any one of three colours different with Area *C*.
Therefore, painting area *A* have 4 methods, painting area *B* have 3 methods, painting area *C* have 2 methods and painting area *D* have 3 methods, there are a total of $4\times3\times2\times3=72$ colouring methods.

Answer：072

25. There is a three-digit number greater than 200 that is a multiple of 5 and 9. Interchanging the first two digits forms a new three-digit number larger than the original. Interchanging the last two digits forms another three-digit number which is a multiple of 2 and smaller than the original. What is the sum of all those possible three-digit numbers?

【Suggested Solution 1】

This kind of problem is to find all possible numbers. Taking each piece into consideration, the other numbers can be found one by one.
Let the three-digit number is $\overline{abc}$. From it is a multiple of 5, we have the digit *c* is either 0 or 5. From interchanging the first two digits the new three-digit number is larger than the original number, we have $b>a$. From interchanging the last two digits the new three-digit number is smaller than the original number and it is a multiple of 2, we have $b>c$ and *b* is even. From it is a multiple of 9, we have $a+b+c=9$ or 18.
If $c=0$, then $a+b=9$ and $2b>a+b=9$. Since *b* is even and $a\geq2$, then $b=6$, $a=3$. From here we got the three-digit number can be 360.
If $c=5$, then $a+b=4$ or 13. Since $b>c$, the only possible is $a+b=13$ and $2b>a+b=13$. Since *b* is even, we have $b=8$, $a=5$. From this we got the three-digit number is 585.
Hence, the sum of all possible three-digit numbers is $360+585=945$.

【Suggested Solution 2】

Let the three-digit number is $\overline{abc} > 200$, it is multiple of 5 and 9, so the possible numbers are 225, 270. 315, 360, 405, 450, 495, 540, 585, 630, 675, 720, 765, 810, 855. 900, 945, 990.

From interchanging the first two digits the new three-digit number is larger than the original number, we have $b > a$. From interchanging the last two digits the new three-digit number is smaller than the original number and it is a multiple of 2, we have $b > c$ and b is even.

Only 360 and 585 fit the condition. Hence, the sum of all possible three-digit numbers is $360 + 585 = 945$.

Answer：945